

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics – III

Subject Code : 4TE03EMT1

Branch: B. Tech (All)

Semester : 3

Date : 25/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) One of the Dirichlet's condition is function $f(x)$ should be
(A) single valued (B) multi valued (C) real valued (D) none of these
- b) Fourier expansion of an even function $f(x)$ in $(-\pi, \pi)$ has
(A) only sine terms (B) only cosine terms
(C) both sine and cosine terms (D) none of these
- c) In the Fourier series expansion of $f(x) = |x|$ in $(-\pi, \pi)$, the value of b_n equal to
(A) 0 (B) π (C) 2π (D) $\frac{\pi}{2}$
- d) Laplace transform of $t^2 e^{-3t}$ is
(A) $\frac{\sqrt{2}}{(s+3)^2}$ (B) $\frac{3!}{(s+3)^2}$ (C) $\frac{2!}{(s+3)^2}$ (D) $\frac{2!}{(s+3)^3}$
- e) Laplace transform of $\frac{\sin t}{t}$ is
(A) $\cot^{-1} \frac{1}{s}$ (B) $\tan^{-1} s$ (C) $\tan^{-1} \frac{1}{s}$ (D) $\sin^{-1} s$
- f) $L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right]$ is
(A) $\frac{t \cos at}{2a}$ (B) $\frac{t^2 \sin at}{2a}$ (C) $\frac{1}{2a^3} (\sin at - at \cos at)$ (D) $\frac{t \sin at}{2a}$
- g) $\frac{1}{(D-2)(D-3)(D-4)} (e^{4x} + e^{2x})$ equal to
(A) $4x(e^{2x} + e^{4x})$ (B) $2(e^{2x} + e^{4x})$ (C) $2x(e^{2x} + e^{4x})$ (D) none of these
- h) The P. I. of $(D^2 + 1)y = \cosh 3x$ is
(A) $\frac{1}{10} \cosh 3x$ (B) $\frac{1}{10} \sinh 3x$ (C) $\frac{1}{5} \cosh 3x$ (D) none of these
- i) The C.F. of the differential equation $(D^3 + 2D^2 + D)y = x^2$ is



- (A) $y = c_1 + (c_2x + c_3)e^{2x}$ (B) $y = c_1 + (c_2 + c_3x)e^{-x}$ (C) $y = c_1 + (c_2x + c_3)e^x$
 (D) none of these
- j) The general solution of the equation $z = px + qy + p^2q^2$ is
 (A) $z = ax + by + c$ (B) $z = ax + by + a^2 + b^2$ (C) $z = ax + by - a^2b^2$
 (D) $z = ax + by + a^2b^2$
- k) The solution of the differential equation $(1+y)p + (1+x)q = z$ is
 (A) $F\left(x(y-z), \frac{x+y+z}{2}\right) = 0$ (B) $F\left(y(x-z), \frac{x+y+z}{2}\right) = 0$
 (C) $F\left(z(y-x), \frac{x+y+z}{2}\right) = 0$ (D) None of these
- l) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is
 (A) $z = f_1(y+x) + f_1(y-x)$ (B) $z = f_1(y+x) + f_2(y-x)$
 (C) $z = f_2(y+x) + f_2(y-x)$ (D) $z = f(x^2 - y^2)$
- m) The order of convergence in Newton-Raphson method is
 (A) 2 (B) 3 (C) 0 (D) None of these
- n) The order of convergence in Bisection method is
 (A) zero (B) linear (C) quadratic (D) None of these

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) Perform the five iteration of the Bisection method to obtain a root of the equation (5)
 $f(x) = \cos x - xe^x$.
- b) Find the root of the equation $\cos x - 3x + 1 = 0$ correct to three decimal positions (5)
 using False position method.
- c) Evaluate: $L(t e^{2t} \cos 3t)$ (4)
- Q-3 Attempt all questions (14)**
- a) Expand $f(x)$ in Fourier series in the interval $(0, 2\pi)$ if (5)
 $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$ and show that $\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$.
- b) Obtain Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ (5)
- c) Given that one root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Find the root correct to 3 significant digits using Secant method. (4)
- Q-4 Attempt all questions (14)**
- a) Using Laplace transform method solve: (5)
 $\frac{d^4 y}{dt^4} - k^4 y = 0, \quad y(0) = y'(0) = y''(0) = 0, y'''(0) = 1, (k \neq 0)$



b) Using convolution theorem, evaluate $L^{-1} \left\{ \frac{s}{(s^2 + 4)^2} \right\}$. (5)

c) Solve: $pz - qz = z^2 + (x + y)^2$ (4)

Q-5

Attempt all questions

(14)

a) Evaluate: $L^{-1} \left[\frac{s+2}{(s+3)(s+1)^3} \right]$ (5)

b) Solve: $(D^2 + 5D + 4)y = x^2 + 7x + 9$ (5)

c) Solve: $2 \frac{\partial^2 z}{\partial x^2} - 5 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = 5 \sin(2x + y)$ (4)

Q-6

Attempt all questions

(14)

a) Solve: $(D^2 - 1)y = \cosh x \cos x$ (5)

b) Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$. Hence deduce (5)

that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$.

c) Solve: $L \left(\frac{e^{-at} - e^{-bt}}{t} \right)$ (4)

Q-7

Attempt all questions

(14)

a) Solve by the method of variation of parameters: $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$ (5)

b) Solve: $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$ (5)

c) Solve: $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 2y$ (4)

Q-8

Attempt all questions

(14)

a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, given (7)

$u(x, 0) = 6e^{-3x}$

b) The following table gives the variations of periodic current $i = f(t)$ amperes over a period T sec. (7)

t (sec) :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
i (A) :	1.98	1.30	1.05	1.30	-0.88	-0.5	1.98

Show, by harmonic analysis, that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

